

APPLICATION

FOR

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TITLE: PROVIDING A FAST ADDER FOR PROCESSOR  
BASED DEVICES

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PROVIDING A FAST ADDER FOR PROCESSOR BASED DEVICES

Background

This invention relates to the field of data processing. More particularly, this invention relates to digital adder circuits used within data processing systems.

Addition is one of the most important arithmetic operations to optimize as it is frequently performed within data processing systems. A problem with producing high speed adder circuits is that the high order bits of the result are logically and physically dependent upon the carry out values from the low order bits. In other words, the carry out from the top bit of the adder is a function of every input bit. Consequently, addition operations tend to be relatively slow.

Considerable effort has been expended for decades to address the problem in order to design and develop adder circuits that are capable of operating at high speed. To that end, various addition algorithms have been developed that include adders using maximally parallel-prefix circuits. While such maximally parallel-prefix adders are a great improvement over prior designs, given the extreme focus on improving processor cycle times, there is a continuing need to reduce the time required to produce a result from an add operation.

### Brief Description of the Drawings

Figure 1 illustrates a two-input-pair carry evaluation circuit.

5        Figure 2 illustrates a 16-bit carry binary number evaluating system incorporating the carry evaluation circuits of Figure 1.

Figure 3 illustrates a three-input-pair carry evaluation circuit.

10       Figure 4 illustrates a 9-bit carry binary number evaluation system incorporating the carry evaluation circuits of Figure 3.

Figure 5 illustrates the interconnections between four-input-pair carry evaluation circuits being used to  
15       calculate the most significant bit of a 32-bit carry binary number.

Figure 6 corresponds to Figure 5 except that the second most significant bit is being evaluated;

Figure 7 illustrates a 4-input pair carry evaluation  
20       circuit.

Figure 8 illustrates a 16-bit adder using 4-input pair carry evaluation circuits.

Figures 9 and 10 illustrate CMOS circuit implementations of 4-input pair carry evaluating circuits.

25       Figure 11 illustrates a conventional adder circuit.

Figure 12 illustrates an adder circuit incorporating 4-input pair carry evaluating circuits.

Figure 13 illustrates a single bit version of the adder of Figure 12.

5        Figure 14 illustrates an improved adder according to one embodiment of the present invention.

Figure 15 illustrates an 80-bit adder according to one embodiment of the present invention.

10       Figure 16 illustrates an adder according to the present invention coupled to a microprocessor.

#### Detailed Description

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The general concept of carry arbitration will be described to aid in the understanding of the invention. In the general case, the carry  $c_{i+1}$  is evaluated by adding two  
15    1-bit binary numbers  $a_i$  and  $b_i$ . There are two general cases defined by the values of  $a_i$  and  $b_i$ . The first case, where there is an output carry request, arises when both operand bits are equal. A 1-carry request occurs if both inputs are 1, whereas a 0-carry request if both inputs are 0. The  
20    second case, where there is no output carry request, arises when the operand bits have different values. See Table 1 in which the letter u indicates there is no output carry request.

TABLE 1

$a_i b_i$	$C_{i+1}$
0 0	0
1 1	1
0 1	u
1 0	u

One input pair  $(a_i, b_i)$  may or may not make a carry request. If two input pairs  $(a_i, b_i)$  and  $(a_j, b_j)$  are used, two carry requests may occur at the same time. Therefore,  
 5 it is necessary to arbitrate these two carry requests. It is of note that  $i$  and  $j$  relate to two adjacent bits (at the first level) or blocks of bits (at subsequent levels) in the calculation, thus if we arbitrating between carry requests relating to previously arbitrated blocks of 3  
 10 bits, then  $i=j+3$ .

Figure 1 shows a 2-input-pair carry arbiter (carry evaluation circuit). The input pair  $(a_i, b_i)$  can make a non-maskable carry request (non-maskable has the meaning that this request must always be acknowledged by the output  
 15 carry  $C_{i+1}$ . The input pair  $(a_j, b_j)$  can make a maskable carry request (maskable has the meaning that this request may be masked by a non-maskable carry request). Only when there is no non-maskable carry request from the input pair  $(a_i, b_i)$ , is a maskable carry request from the input pair  
 20  $(a_j, b_j)$  acknowledged by the output carry  $C_{i+1}$ . This is illustrated in Table 2 below.

TABLE 2

$a_i b_i$	$a_j, b_i$	$C_{i+1}$
0 0	- -	0
1 1	- -	1
0 1 (or 1 0)	0 0	0
0 1 (or 1 0)	1 1	1
0 1 (opr 1 0)	0 1 (or 1 0)	u

The output carry  $c_{i+1}$  1 can be encoded using two wires  $(v_i, w_i)$  as illustrated in Table 3 below.

5

Table 3

$C_{i+1}$	$V_i, W_i$
0	0 0
1	1 1
u	0 1
u	1 0

The signals on the two wires constitute the carry production control signal. The following equations satisfy the conditions illustrated in Tables 2 and 3:

$$\begin{aligned}
 V_i &= a_i b_i + (a_i + b_i) a_j & (1) \\
 W_i &= a_i b_i + (a_i + b_i) b_j
 \end{aligned}$$

A 16-bit fast carry computation using 2-input-pair carry arbiters is shown in Figure 2, which illustrates that the scheme is regular. Each node in Figure 2 is a 2-input-bit carry arbiter. Depending on its inputs, each node can be considered to "vote" on the result to be passed up to the next level in the circuit. The node can indicate a carry generate (1, 1) (a vote yes), a carry kill (0, 0) (a vote no) or a carry propagate (0, 1) or (1, 0) (an abstention).

At the bottom level, this arbitration is carried out

between bits of the input operands, and at higher level between the results of previously determined arbitrations.

A 3-input-pair carry arbiter is shown in Figure 3. The input pair  $(a_i, b_i)$ , can make a non-maskable carry request.

5 The input pairs  $(a_j, b_j)$  and  $(a_k, b_k)$  can both make a maskable carry request at the same time. However, the input pair  $(a_j, b_j)$  has priority over the input pair  $(a_k, b_k)$ . Only when there is not a non-maskable carry request from the input pair  $(a_i, b_i)$  and no maskable carry from the input  
 10 pair  $(a_j, b_j)$ , is a maskable carry request from the input pair  $(a_k, b_k)$  acknowledged by the output carry  $c_{i+1}$  as illustrated in Table 4 below.

TABLE 4

$a_i, b_i$	$a_j, b_j$	$a_k, b_k$	$c_{i+1}$
0 0	- -	- -	0
1 1	- -	- -	1
0 1 (or 1 0)	0 0	- -	0
0 1 (or 1 0)	1 1	- -	1
0 1 (or 1 0)	0 1 (or 1 0)	0 0	0
0 1 (or 1 0)	0 1 (or 1 0)	1 1	1
0 1 (or 1 0)	0 1 (or 1 0)	0 1 (or 1 0)	u

15 The following equations satisfy Tables 3 and 4:

$$V_i = a_i b_i + (a_i + b_i) (a_j b_j + (a_j + b_j) a_k) \quad (3)$$

$$V_i = a_i b_i + (a_j + b_i) (a_j b_j + (a_j + b_j) b_k)$$

Figure 4 shows a 9-bit carry computation using 3-  
 20 input-pair carry arbiters, which results in only two layers of logic and hence high-speed carry generation. The addition of n-bit binary numbers using 3-input-pair carry

arbiters can be performed in time proportional to  $O(\log_3 n)$ , and therefore is more efficient than using 2-input-pair carry arbiters where the computation time is  $O(\log_2 n)$ .

Using a similar approach to 2- or 3-input-pair carry arbiters, carry arbiters with any numbers of input pairs can be derived. However, carry arbiters with more than 4 input pairs are not usually of interest. Firstly, too many series transistors are needed to implement these arbiters, which leads to inefficient CMOS designs. Secondly, the arbiter cell layout may become too large for the bit slice of a datapath.

Figure 5 shows a part of a 32-bit adder design that generates the carry for the 32nd bit. 4-input-pair carry arbiters are used in the first and second rows (from the bottom), whereas 2-input-pair arbiters are employed in the third row. The carry computation goes through only three logic layers.

Figure 6 shows the part of the circuit that generates the 31st bit carry. Analogous circuit (interconnections) to those shown in Figures 7 and 8 are used for the other bits of the carry result. Once a carry bit has been determined (i.e. the carry-in and structure result a generate or a kill, with propagate not being possible at that point), then a single signal wire may be used to pass that result to higher levels.



The final row is a sum circuit that operates to XOR the input operands and the carry result.

The carry out from the adder of Figures 5 and 6 may be achieved by expanding the three rows of carry arbiters to include a bit 32 and then putting a two bit arbiter in position 32 in the fourth row with inputs from positions 0 and 32 of the previous row. Alternatively, the circuit at position 31 in the fourth row (of adder circuits) may be changed into a full adder generating both sum and carry outputs rather than just the sum output of the XOR circuits.

As soon as  $v_i$  and  $w_i$  are equal (meaning that the carry has been generated), only single-rail signals need to be routed instead of dual-rail signals. This results in a significant reduction of chip area, especially in the third row where more room is needed to accommodate signals crossing from the least significant bits to the most significant bits. Thus, the resulting adder is quite compact.

As an example of the use of the above technique, the design of an 80-bit high-speed adder with a moderate chip area will now be considered.

The carry  $c_i$  is evaluated by adding two 1-bit numbers  $a_i$  and  $b_i$  as shown in Table 1. There are two general cases defined by the values  $a_i$  and  $b_i$ . The first case, where there is a carry request, arises when both operand bits are

equal. A 1-carry request occurs if both inputs are 1,  
whereas a 0-carry request if both inputs are 0. The second  
case, where there is no carry request, arises when the  
operand bits have different values. The letter u indicates  
5 there is no carry request.

We introduce the concept of carry arbitration by  
taking a four-way carry arbiter as shown in Figure 7 as an  
example. Any input pair  $a_i$  and  $b_i$  ( $0 \leq i \leq 3$ ) can make a  
carry request and hence two or more carry requests may  
10 occur at the same time. Therefore, it is necessary to  
arbitrate these carry requests. The input pair  $(a_3, b_3)$  can  
make a non-maskable carry request, where non-maskable means  
that a carry request from the input pair  $(a_3, b_3)$  must  
always be acknowledged by the output "c". The other three  
15 input pairs  $(a_2, b_2)$ ,  $(a_1, b_1)$  and  $(a_0, b_0)$  can make maskable  
carry requests, where maskable means that carry requests  
from these three input pairs may be masked by the input  
pair  $(a_3, b_3)$ . The input pair  $(a_2, b_2)$  has higher priority  
than the input pairs  $(a_1, b_1)$  and  $(a_0, b_0)$ . The input pair  
20  $(a_0, b_0)$  has the lowest priority.

Only when there is no non-maskable carry request from  
the input pair  $(a_3, b_3)$  is a maskable carry request from the  
input pair  $(a_2, b_2)$  acknowledged by the output c. Only when  
there is no non-maskable carry request from the input pair  
25  $(a_3, b_3)$  and no maskable carry request from the input pair  
 $(a_2, b_2)$  is a maskable carry request from the input pair

( $a_1, b_1$ ) acknowledged by the output  $c$ . Only when there are no carry requests from the input pairs ( $a_3, b_3$ ), ( $a_2, b_2$ ) and ( $a_1, b_1$ ) is a carry request from the input pair ( $a_0, b_0$ ) acknowledged by the output  $c$ . Table 5 outlines the truth table required to implement four-way carry arbiters.

TABLE 5

$a_3, b_3$	$a_2, b_2$	$a_1, b_1$	$a_0, b_0$	$c$
00	-	-	-	0
11	-	-	-	1
01 or 10	00	-	-	0
01 or 10	11	-	-	1
01 or 10	01 or 10	00	-	0
01 or 10	01 or 10	11	-	1
01 or 10	01 or 10	01 or 10	00	0
01 or 10	01 or 10	01 or 10	11	1
01 or 10	01 or 10	01 or 10	01 or 10	u

Using the same approach, carry arbiters with any number of ways may be derived. The carries may be generated quickly by using carry arbiters combined into a tree structure that exploits the associativity of the carry computation.

Figure 8, for example, illustrates 16-bit carry computation base on the carry arbitration. The solid dots represent carry arbiters. The addition of  $n$ -bit numbers using  $m$ -way carry arbiters can be performed in a time proportional to  $O(\log_m n)$ .

Theoretically, the more inputs each carry arbiter handles, the faster the carries are generated. However, as discussed earlier, carry arbiters with more than four ways

are not usually of practical interest. Four-way carry arbiters and their dynamic CMOS implementation may be chosen because they may achieve advantageous results for a 80-bit design. Other designs, such as 32-bit adders, favor three-way carry arbiters.

Motivated by the dual-rail data encoding used in self-timed design, the carry request out  $c$  can be encoded using two wires ( $aa$ ,  $bb$ ) as shown in Table 6, below.

TABLE 6

$c$	$aa, bb$
0	0 0
1	1 1
$u$	0 1
$u$	1 0

Equations 6 and 7 give the behavior defined by Tables 5 and 6.

$$aa = a_3b_3 + (a_3 + b_3) (a_2b_2 + (a_2 + b_2) (a_1b_1 + (a_1 + b_1)a_0)) \quad (6)$$

$$bb = a_3b_3 + (a_3 + b_3) (a_2b_2 + (a_2 + b_2) (a_1b_1 + (a_1 + b_1)b_0)) \quad (7)$$

Figure 9 shows a direct dynamic CMOS implementation of the four-way carry arbiter according to the above equations. The operation of the circuit is such that the nodes  $n1$  and  $n2$  are precharged high when the inputs  $a_3$  and  $b_3$  are low during the reset phase of the control handshake and will conditionally discharge during the evaluation phase. The buffers are used to maintain drive strength.

Figure 10 gives a modified version of the four-way carry arbiter. We assume here that every input pair ( $a_i$ ,  $b_i$ )

takes one of the three values (0 0), (1 1) and (1 0), and (0 1) has already been transformed to (1 0). The reasons are twofold. Firstly, it is easy to layout the modified circuit. Secondly and more importantly, the outputs aa and bb have new meaning. If the outputs aa and bb have different values, this means there are no carry requests from the inputs as described previously. Of course, one may design the adder to use the value of (0 1) instead of (1 0) and, in that case, convert an input pair value of (1 0) to (0 1).

However, we can take another view of a four-way carry arbiter. If we consider a four-way carry arbiter as a carry generation circuit for a 4-bit addition, then one of the outputs aa and bb can be viewed as the carry out generated with a zero carry-in and the other is with a one carry-in. The direct implementation does not distinguish which is the carry out generated with a zero carry-in and which with a one carry-in. In the modified circuit, the output aa is the carry out generated with a one carry-in and the output bb as the carry out generated with a zero carry-in. This may result in a significant reduction of chip area.

However, the use of the modified implementation needs the input conversion from (0 1) to (1 0). Fortunately this conversion is straightforward. It consists of one 2-input NAND and one 2-input NOR gate per bit. For practical reasons, gates are normally necessary anyway to isolate the

signals from the main input buses. The difference here is that NAND and NOR gates are used instead of inverters. If two input buses are designed using a precharge structure, the outputs after NAND and NOR gates are naturally low  
5 (required in the dynamic implementation) when the buses are precharged high. Furthermore, these NAND and NOR gates can be reused for logic operations in an ALU design.

Consider first a conventional approach to high-speed adder design. Figure 11 shows a conventional adder design  
10 using the carry select scheme. The inputs are divided into d-bit groups. Two adders are needed per group. One is an adder with a zero carry-in and the other with a one carry-in. The carry generator is responsible for generating the boundary carries for all groups, which are then used to  
15 select the appropriate sum using a multiplexer. A design decision must be made to choose appropriate groups in order to balance the delays of both the carry generator and the group adders. If the group adders are made too long, then the decreasing delays in the carry generator are exceeded  
20 by the increasing delays of the group adders. If the group adders are made too short, the logic depth of the carry generator increases and its delay determines the total adder delay.

A block diagram of an improved 80-bit adder is shown  
25 in Figure 12. The whole adder is visualized (but not divided) as consisting of five 16-bit groups. The first row

is the conversion circuit, which contains 2-input NAND and NOR gates 1201-1209. The second and third rows are four-way arbiters that produce carries within each group and have the form discussed previously. The fourth row, 1211-1219, produces two intermediate sums with a zero carry-in and a one carry-in. The final row includes multiplexers which select the final sum result and three carry arbiters which generate the boundary carries  $c_{16}$ ,  $c_{32}$ ,  $c_{48}$  and  $c_{64}$ . The carries of the 16 least significant bits have already been generated after two rows of the carry computation. Compared with the conventional carry-select scheme, the need for group adders has been eliminated. The two intermediate sums are elegantly generated within the carry generation tree. This may result in a significant reduction of chip area, especially when the groups are made to be long, since group adders also need some mechanisms for carry computation.

It is worth noting that only single-rail signals need to be routed (instead of dual-rail signals) if the signals  $aa$  and  $bb$  are known to be equal (meaning that the carry has been generated, as either a 1-carry or a 0-carry request).

By exploiting the properties of associativity of the logic equation governing carry generation, the adder illustrated in Figure 12 represents a significant improvement over traditional adder designs. However, the propagation delays through the adder of Figure 12 may still be undesirable.

To better understand the propagation delays through the adder of Figure 12, reference will now be made to Figure 13. Figure 13 illustrates a single bit adder version of the adder in Figure 12 where the rows in Figure 5 13 correspond to like referenced rows in Figure 12. As described before, the 1<sup>st</sup> row includes a conversion stage 1301. The second and third rows include 4-way carry evaluators 1303 and 1305 respectively. The fourth row includes a XOR 1309 logic stage.

10 The 5<sup>th</sup> row has been split into its constituent parts: a 4-way carry evaluator 1307 logic stage and a multiplexer 1311 logic stage. As before, row 4 produces two intermediate sums, res\_0 and res\_1, with a zero carry-in and a one carry-in respectively. The symbol "1" represents 15 that the node is always logic "1" if two carry outputs are different. The symbol "0" represents that the node is always logic "0" if two carry outputs are different. The fifth row multiplexer 1311 selects the final sum result. The 4-way carry evaluator 1307 may be a carry-select 20 evaluator circuit.

The total propagation delay " $T_{pt}$ " through all the adder logic stages, row 1 - row 5, is the total of the individual propagation delays " $T_p$ " through each of the first three rows plus the longest of the propagation times associated with 25 row 4 and row 5. The propagation time through the 4-way carry evaluator 1307 and the multiplexer 1311 may be



considerably longer than the propagation time through the XOR 1309 and the multiplexer 1311. Therefore the total propagation time can be represented as:

$$T_{pt} = T_p \text{ 1}^{st} \text{ row} + T_p \text{ 2}^{nd} \text{ row} + T_p \text{ 3}^{rd} \text{ row} + T_p \text{ 5}^{th} \text{ row}.$$

5 In essence, the 4<sup>th</sup> and 5<sup>th</sup> rows operate in parallel. As can be seen from the above equation, the XOR logic stage 1309 is not in the critical timing path and the propagation time through this stage is considered a "slack path". The slack path time "S<sub>pt</sub>" is simply the difference in the  
10 longest propagation path and the shortest propagation path with regards to the 4<sup>th</sup> row and can be expressed:

$$S_{pt} = ((T_p \text{ 1307} + T_p \text{ 1311}) - (T_p \text{ 1309} + T_p \text{ 1311}))$$

15 Figure 14 illustrates one embodiment of the present invention where the functionality of the conversion stage 1301 has been moved from row 1 (relative to Figure 12) to before the XOR logic stage 1309 in row 4. This change moves the propagation delay from row 1, which has now been  
20 eliminated, to the slack path in row 4. The symbol "u" represents that the node may be either logic "1" or logic "0" if two of the carry outputs are different. Knowing which carry output is "0" and which one is "1" is of no consequence for carry-generating circuits such as 1303 and  
25 1305. However, knowing which carry output is "0" and which is "1" does matter for generating the intermediate results res\_0 and res\_1 by the XOR stage 1309. By placing the

conversion stage 1301 before the XOR stage 1309, the same logic equation is achieved by the adder in Figure 14 as by the adder of Figure 13.

5 In Figure 14, given that  $T_p$  1<sup>st</sup> row is 0 (row 1 has been eliminated), for the first case where the propagation delay  $T_p$  1307 +  $T_p$  1311 is equal to or greater than  $T_p$  1301 +  $T_p$  1309 +  $T_p$  1311, then the total propagation delay  $T_{pt}$  is reduced to:

$$T_{pt} = T_p \text{ 2<sup>nd</sup> row} + T_p \text{ 3<sup>rd</sup> row} + + T_p \text{ 5<sup>th</sup> row}.$$

10

In this case, the propagation delay through the adder may be reduced by the propagation time through row 1 ( $T_p$  1301) relative to the adder of Figure 13.

15 For the second case where the propagation time through the path  $T_p$  1301 +  $T_p$  1309 +  $T_p$  1311 is greater than through the propagation path  $T_p$  1307 +  $T_p$  1311, then the total propagation delay  $T_{pt}$  for the adder in Figure 14 is reduced by the slack path time  $S_{pt}$  relative to the  $T_{pt}$  for the adder of Figure 13. In this case,  $T_{pt}$  becomes:

20  $T_{pt} = T_p \text{ 2<sup>nd</sup> row} + T_p \text{ 3<sup>rd</sup> row} + T_p \text{ 4<sup>th</sup> row} (T_p \text{ 1301} + T_p \text{ 1309}) + T_p \text{ 5<sup>th</sup> row} (1311).$

So in either the first or the second case, the adder of Figure 14 may have a reduced propagation time  $T_{pt}$  relative to the adder of Figure 13.

25 Figure 15 illustrates a block diagram of an 80-bit adder according to another embodiment of the invention. Again, this adder may be logically visualized (but not

physically divided) into rows. Relative to the adder of Figure 12, the first row is eliminated by merging the functionality of the 2-input AND and OR gates 1201 - 1209 into the 4<sup>th</sup> row (relative to Figure 12) with the XOR stages 1211 - 1219 respectively to form a combined third row. The functionality of the combined third row is as described above with respect to Figure 14.

As described in association with Figure 14, by moving the AND and OR gates 1201 - 1209 into the slack path of the 4<sup>th</sup> row (with respect to the adder of Figure 12), the total propagation time through the adder may be reduced. The critical timing path may be reduced from four rows to three rows.

Current process technologies may allow CMOS circuits operating at over 1 GHZ to accommodate about ten gates within a pipeline stage. This indicates that the adder of Figure 15 may have an improved performance over the adder of Figure 12 by about 10%. This improvement may be achieved without additional hardware overhead relative to the adder of Figure 12.

In some embodiments of the present invention as shown in Figure 16, the high speed adder 1601 may be coupled to a microprocessor "CPU" 1603 with busses 1607. The adder may provide on-chip addition hardware for an arithmetic logic 1605 unit inside the CPU. In other embodiments, the high speed adder 1601 may be a co-processor external to the

microprocessor 1603 and may be coupled to the  
microprocessor with busses 1607.

Although illustrative embodiments of the invention  
have been described in detail herein with reference to the  
5 accompanying drawings, it is to be understood that the  
invention is not limited to those precise embodiments, and  
that various changes and modifications can be effected  
therein by one skilled in the art without departing from  
the scope and spirit of the invention as defined by the  
10 appended claims.

The present invention has been described with respect  
to a limited number of embodiments; those skilled in the  
art will appreciate numerous modifications and variations  
therefrom. It is intended that the appended claims cover  
15 all such modifications and variations as fall within the  
true spirit and scope of this present invention.

What is claimed is: